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Power Networks Planning Using Mixed-Integer Programming

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Abstract. The main objective of this paper is to formulate a power distribution planning model to solve the optimal sizing, timing, and location of distribution substation and feeder expansion problems simultaneously. The objective function of the model represents the present worth of costs of investment and energy losses of the system which take place throughout the planning time horizon. The objective function (worth of cost) is minimized subject to Kirchhoff's Current Law, power capacity limits, voltage drop, and logical constraints, by using a mixed-integer programming algorithm.

Keywords. Planning Models; Kirchhoff's Laws; Mixed-Integer; Static-Load; Dynamic-Load; XLPE Cable (cross-linked polyethylene insulated cable); Optimization.

Introduction

During the last three decades, several power system distribution planning models have been presented [1-13]. The approaches of those planning models can be classified into four categories: Static-load subsystem, static-load total-system, dynamic-load subsystem, and dynamic-load total-system. Static approach considers the distribution system load as constant with time, but a dynamic approach includes load-time variations in the planning process. A subsystem approach describes only a portion of the distribution system (substation or feeder), while a total-system approach considers both substations and feeders in the planning formulation.

The main objective of this paper is to formulate a power distribution planning model in a way to overcome some of the limitations that occur in the existing distribution planning models. The most important of these limitations are summarized as follows

(i) Due to the complexity of the problem, various approximations are necessary in the planning models. For instance, some of the models do not include feeder fixed cost in the total-cost formulation, while others have either linearized or ignored the nonlinear cost function of the distribution substations and feeder segments, as it is apparent in the references [1-13]. In the planning process, the fixed cost component of the feeder segment cannot be neglected without affecting the overall solution validity. Also the cost of actual feeder losses is nonlinear with respect to the level of feeder loading. The linear approximation of the total cost, leads to a near optimal but not the optimal planning solution.

(ii) Many of the existing planning models do not include voltage constraints into the planning problem formulation [1-3, 5, 7]. When voltage constraints are not incorporated into the planning process, excessive voltage drops beyond the maximum acceptable limits can appear in the final solution over some paths of the distribution network, resulting in an impractical network configuration.

(iii) Some of the existing distributions planning models do not consider the time factor [1-4]. Such models solve the planning problem in the static (one-time-step) mode. Over a long period of planning, this mode might not lead to an optimal solution. True optimality can be assessed only over a time period of specific duration, but not in the static manner.

Model Architecture

The work is applied to the 380/110 kV network derived from the large scale network of the Western Region in Saudi Arabia shown in Fig.1. This network is classified as a distribution network [14]. The data was provided in May 2004. The network has two existing transformer stations, a potential substation, thirty two feeder routes, and eleven demand points. The existing substations have initially 3 units of capacity in the range 250 to 500 MVA and can adopt a total of 4 to 5 units. The potential substation can have a total of 2 to 4 units of capacity 90 MVA.

The primary three phase feeders have two types of conductors, 630 mm2 XLPE , 110kV, YJLV “Al-Core” and 630 mm2 XLPE , 110kV, YLV “Cu-Core” cable.

The Developed Model

A mixed-integer model is developed to solve the optimal sizing, timing, and location of substation and feeder expansion problem simultaneously. This developed model allows for the inclusion of the explicit constraints of voltage drop in its formulation. The cost function is used to formulate the objective function that is to be minimized in the optimization process.

The Objective Function

$$F=\sum_{t=0}^{T_p-1}\sum_{i\in N_s}\sum_{k'\in N_{k'}}(C_{si})_{k'}(X_{it})_{k'}+\sum_{t=0}^{T_p-1}\sum_{(i,j)\in N_f}\sum_{k\in N_k}(C_{fij})_k(Y_{ijt})_k+\sum_{t=0}^{T_p-1}\sum_{(i,j)\in N_f}\sum_{k\in N_k}(C_{lij})_k[(P_{ijt})_k-(P_{jit})_k]^2$$
$$+\sum_{t=0}^{T_p-1}\sum_{(i,j)\in N_{fe}}(C_{lij})_e[(P_{ijt})_e-(P_{jit})_e]^2 \qquad (1)$$

where:

$(C_{si})_{k'}$ is the fixed cost of potential station of size *k’*, associated with node *i*.

$(X_i)_{k'}$ is a binary variable that is equal to one if a substation with size *k’*, associated with node *i*, is to be built . Otherwise, it is equal to zero.

$(C_{fij})_k$ denotes the fixed cost of potential feeder segment of size *k*, associated with route*(i,j)*

$(Y_{ij})_k$ is a binary variable that is equal to one if a feeder segment with size *k*, associated with route *(i,j)*, is to be built . Otherwise, it is equal to zero.

C_{lij} is the loss cost coefficient.

$(P_{ij})_k$ is the forward power flow through route (*i,j*) associated with a proposed feeder size *k* in MVA.

$(P_{ji})_k$ is the reverse power flow through route (*i,j*) associated with a proposed feeder size *k* in MVA.

$(P_{ij})_e$ is the forward power flow through route (*i,j*) referred to an existing feeder in MVA.

$(P_{ji})_e$ is the reverse power flow through route (i,j) referred to an existing feeder in MVA.

T_p is the planning period.

N_s is the set of nodes associated with substations.

N_f is the set of proposed feeder routes (between nodes) to be built in the planning period.

N_{fe} is the set of paths (between nodes) associated with existing feeders in the initial network to be expanded.

$N_{k'}$ proposed substation sizes to be built in the planning period.

N_k proposed feeder sizes to be built in the planning period.

1. ***Power Demand Constraints***

The power demands of the load centers must be met throughout the planning period. Kirchhoff's Current Law is applied to each year therefore:

$$\sum_{j \in N_i} \sum_{k \in N_k} (P_{ijt})_k - (P_{jit})_k = D_{it} \quad , \quad \forall\, i \in N \ , \tag{2}$$

where:

N is the sum of the existing and future node sets.

N_i is the set of nodes that are connected to node i or that are proposed to be connected to node i .

D_{it} is the peak power demand at node i at time t.

2. ***Power Capacity Constraints***

Limits for the maximum capacity of the power that can be carried by each feeder and supplied by each substation are specified according to their proposed sizes.

For a future feeder on route (i,j):

$$0 \le (P_{ijt})_k \le (U_{ij})_k \sum_{\tau=0}^{t} (Y_{ij\tau})_k \quad , \quad \forall (i,j) \in N_f \;\; and \;\; \forall\, k \in N_k , \tag{3}$$

$$0 \le (P_{jit})_k \le (U_{ij})_k \sum_{\tau=0}^{t} (Y_{ij\tau})_k \quad , \quad \forall (i,j) \in N_f \;\; and \;\; \forall k \in N_k \ . \tag{4}$$

For an existing feeder on route (i,j):

$$0 \quad \leq (P_{ijt})_e \leq (U_{ij})_e \quad , \qquad \forall (i,j) \in N_{fe} \ , \tag{5}$$

$$0 \quad \leq (P_{jit})_e \leq (U_{ij})_e \quad , \qquad \forall (i,j) \in N_{fe} \ . \tag{6}$$

For a future substation:

$$0 \leq \sum_{j \in N_i} (P_{ijt})_k - (P_{jit})_k \leq (U_i)_{k'} \sum_{\tau=0}^{t} (X_{i\tau})_{k'} \quad , \quad \forall i \in N_s \ and \quad \forall k' \in N_{k'} . \tag{7}$$

For an existing substation:

$$0 \quad \leq \quad \sum_{j \in N_i} (P_{ijt})_e - (P_{jit})_e \leq (U_i)_e \ , \qquad \forall i \in N_{se} , \tag{8}$$

where:

$(U_{ij})_k$ is the MVA capacity limit of a feeder of size k , associated with route (*i,j*).

$(U_{ij})_e$ is the MVA capacity limit of an existing feeder associated with route (*i,j*).

$(U_i)_{k'}$ is the MVA capacity limit of a substation of size k' , associated with node *i*.

$(U_i)_e$ is the MVA capacity limit of existing substation associated with node *i*.

3. *Logical Constraints*

These are the mathematical constraints linking the planning decision variables for building substations or feeders (and their specified sizes) throughout the planning period. For future substations, the following constraints guarantee that only one substation of a given size can be built at a given location over the planning period:

$$\sum_{\tau=0}^{T_{p-1}} \sum_{k' \in N_{k'}} (X_{i\tau})_{k'} \quad \leq \quad 1 \quad , \quad \forall \ i \in N_s . \tag{9}$$

The total number of future substations can be limited using the following constraint:

$$\sum_{\tau=0}^{T_{p-1}} \sum_{i \in N_s} \sum_{k' \in N_{k'}} (X_{i\tau})_{k'} \quad \leq \quad n_s . \tag{10}$$

For future feeders, the following constraints guarantee that only one feeder of a given size can be built on feeder route (*i,j*) over the planning period:

$$\sum_{\tau=0}^{T_{p-1}} \sum_{k\in N_k} (Y_{ij\tau})_k \leq 1 \quad , \quad \forall\,(i,j)\in N_f . \tag{11}$$

The total number of feeders to be built can be limited by using the following constraint:

$$\sum_{\tau=0}^{T_{p-1}} \sum_{(i,j)\in N_f} \sum_{k\in N_k} (Y_{ij\tau})_k \leq n_f . \tag{12}$$

where:

n_s is the number of elements in set N_s.

n_f is the number of elements in set N_f.

4. *Voltage Drop Constraints*

The percent voltage drop of a feeder over its entire length can be expressed as

$$\%\,\Delta V = [\frac{S}{V^2_{nom}}]\, l\,(R\cos\Phi + X\sin\Phi)\,100, \tag{13}$$

where:

S is the three-phase apparent power in MVA.

l is the length of the feeder in km.

R is the resistance of the feeder in ohm per kilometer.

X is the reactance of the feeder in ohm per kilometer.

$\cos\Phi$ is the power factor.

V_{nom} is the line-line nominal voltage in kV.

The voltage drop constraints are built for the following general cases[13]:

Case 1

Consider the future feeder to be built between demand nodes *i* and *j* on route (*i,j*). The required voltage drop constraints are

$$[(V_i)_t + D_{ijt}] - [(V_j)_t + D_{jit}] = \sum_{k \in N_k} [(P_{ijt})_k - (P_{jit})_k]\, G_{ijkt} \quad , \tag{14}$$

$$0 \le D_{ijt} \le D_{up}\, [1 - \sum_{\tau=0}^{t} \sum_{k \in N_k} (Y_{ij\tau})_k] \quad , \qquad \forall (i,j) \in N_f \;, \tag{15}$$

$$0 \le D_{jit} \le D_{up}\, [1 - \sum_{\tau=0}^{t} \sum_{k \in N_k} (Y_{ij\tau})_k] \quad , \qquad \forall (i,j) \in N_f \quad . \tag{16}$$

$$D_{up} = V_{nom} [\frac{(\% \Delta V_{\max})}{100}] \,. \tag{17}$$

$$V_{low} \le (V_i)_t \le V_{nom} \tag{18}$$

$$V_{low} \le (V_j)_t \le V_{nom} \tag{19}$$

Case 2

Consider the existing feeder (with conductor size k_e) between demand nodes i and j on route (i,j). The required voltage drop constraints are

$$(V_i)_t - (V_j)_t = [(P_{ij})_{k_e} - (P_{ji})_{k_e}]\, G_{ijk_e t} \quad , \quad \forall (i,j) \in N_{fe} \quad , \tag{20}$$

$$V_{low} \le (V_i)_t \le V_{nom} \tag{21}$$

$$V_{low} \le (V_j)_t \le V_{nom} \tag{22}$$

Case 3

Consider the future feeder to be built between a future substation located at node i and a demand node j located on route (i,j). The required voltage drop constraints are

$$[(V_i)_t + D_{ijt}] - [(V_j)_t + D_{jit}] = \sum_{k \in N_k} [(P_{ijt})_k - (P_{jit})_k]\, G_{ijkt} \tag{23}$$

$$0 \le D_{ijt} \le D_{up}\, [1 - \sum_{\tau=0}^{t} \sum_{k \in N_k} (Y_{ij\tau})_k] \quad , \qquad \forall (i,j) \in N_f \tag{24}$$

$$0 \le D_{jit} \le D_{up}\, [1 - \sum_{\tau=0}^{t} \sum_{k \in N_k} (Y_{ijt})_k] \quad , \qquad \forall (i,j) \in N_f \tag{25}$$

$$(V_i)_t = V_{nom}\,[\sum_{\tau=0}^{t} \sum_{k' \in N_{k'}} (X_{i\tau})_{k'}\,] + V_{itF} \quad ,\forall\, i \in N_s\,, \tag{26}$$

$$0 \;\le\; V_{itF} \;\le\; V_{nom}\,[1 - \sum_{\tau=0}^{t} \sum_{k' \in N_{k'}} (X_{i\tau})_{k'}\,] \quad ,\forall i \in N_s\,. \tag{27}$$

If the demand center of the substation has an associated demand that is not zero, then include equation (17), otherwise consider $D_{up} = V_{nom}$.

If the demand center of the substation has an associated demand other than zero, then include equations (21) and (22).

If the demand center of the substation has zero demand, then

$$V_{nom}\,[\sum_{\tau=0}^{t} \sum_{\alpha \in (N_t)_i} \sum_{k \in N_k} (Y_{i\alpha\tau})_k\,] \ge (V_i)_t \tag{28a}$$

$$V_{low}\,[\sum_{\tau=0}^{t} \sum_{k \in N_k} (Y_{i\alpha\tau})_k\,] \le (V_i)_t \qquad , \qquad \forall\, \alpha \in (N_t)_i \tag{28b}$$

Case 4

Consider the future feeder to be built between an existing substation located at node *i* and a demand node *j* located on route (*i,j*). The required voltage drop constraints are

$$V_{nom} - [(V_j)_t + D_{jit}\,] = \sum_{k \in N_k} [(P_{ijt})_k - (P_{jit})_k\,]\, G_{ijkt} \quad , \quad \forall (i,j) \in N_f\,, \tag{29}$$

Also include equations (25), (17a) and (22).

Case 5

Consider the existing feeder (with conductor size k_e) between an existing substation located at node *i* and a demand node *j* located on route (*i,j*). The required voltage drop constraints are

$$V_{nom} - (V_j)_t = [(P_{ijt})_{k_e} - (P_{jit})_{k_e}\,]\, G_{ijk_e t} \quad , \quad \forall (i,j) \in N_{fe} \quad , \tag{30}$$

and equation (22).

Case 6

Consider the existing feeder (with conductor size k_e) between a future substation located at node i and a demand node j located on route (i,j). The required voltage drop constraints are

$$(V_i)_t - (V_j)_t = [(P_{ijt})_{k_e} - (P_{jit})_{k_e}]\, G_{ijk_e t} \quad , \qquad \forall (i,j) \in N_{fe} \quad , \tag{31}$$

together with equations (26), (27), (21) and (22).

where:

$(V_i)_t$ is the line-line voltage at node i at time t.

$(V_j)_t$ is the line-line voltage at node j at time t.

$\%\Delta V_{max}$ is the maximum acceptable voltage drop from each substation to each node in percent

V_{low} is the minimum line-line voltage in kV, such that:

$$V_{low} = V_{nom}\,[1 - \frac{\%\Delta V_{max}}{100}]$$

V_{jtF} is a continuous and nonnegative variable.

D_{ijt} is a continuous and nonnegative deviation or slack variable.

D_{jit} is a continuous and nonnegative deviation or slack variable.

$(N_t)_i$ is the nodes that are proposed to be connected to node i at, or before time t.

G_{ijkt} is a constant, such that:

$$G_{ijkt} = [\frac{R_k\,(\cos\Phi)_t + X_k\,(\sin\Phi)_t}{V_{nom}}]\, l_{ij}$$

R_k is the resistance of feeder of size k in ohm per kilometer.

X_k is the reactance of feeder of size k in ohm per kilometer.

$(\cos\Phi)_t$ is the power factor of the network at time t.

l_{ij} is the length of feeder connecting node i to node j in kilometer.

Application of the Model

In the original network model there are twenty seven feeders in existence to supply the demand at the start of the planning period. One transformer station is to be built within the period of the planning study to meet the growing load demands (110/33 kV distribution stations). A potential transformer station site is selected based on both land availability and proximity to load centers. Several potential feeder routes are chosen on the basis that the growing load demands are supplied by either an existing transformer stations or a new station. The cost of each 110 kV feeder is 453000 dollar/km (assumed a fixed cable cost factor), with a maximum capacity of 81 MVA for 630 mm2 XLPE , 110kV, YJLV "Al-Core" and 88 MVA for 630 mm2 XLPE , 110kV, YLV "Cu-Core" cable. New stations have an estimated cost of six million dollars, and a maximum feasible capacity of 180 MVA. The cost of energy is 0.15 dollar/kWh. Interest rate is assumed to be constant at 10%. Feeder life, power factor and loss-load factor are assumed to be 25 years, 0.85 and 0.21, respectively. The network potential sites for locating the grid transformer stations, the annual peak power demands requirements of the distribution network over the four year period, and the possible routes for 110 kV feeders are given in Tables 1-3.

Table 1. Substation data.

Station	Node	Capacity			Cost			Max. Feeders	Status
		MVA			$ M				
		Size 1	Size 2	Size 3	Size 1	Size 2	Size 3	Size 1-3	
A	10	1500	2000	2500	45	60	75	8	Existing
B	11	750	1000	1250	24	32	40	8	Existing
C	12	180	270	360	6	9	12	8	Potential

The model is used to solve the planning problems using the LINGO 7.0 software package. LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems concisely, solve them, and analyze the solution. The model has been applied to solve various cases as explained in the following paragraphs.

Table 2. Annual peak power demands requirements, MVA.

Node	First year	Second year	Third year	Fourth year
1	45	45	46	47
2	79	80	82	84
3	83	88	93	99
4	28	32	38	44
5	99	105	112	119
6	43	51	60	70
7	82	87	92	98
8	81	83	85	87
9	74	78	82	87
13	50	58	67	77
14	66	75	85	98

Table 3. Feeders data.

Feeder	From Node	To Node	Length km	Capacity MVA	Cost M$	Loss coefficient M$/MVA²	G kV/MVA	Status
1	10	1	7.29	81	3.302	8.74E-07	9.95E-04	Existing
2	10	1	7.29	81	3.302	8.74E-07	9.95E-04	Existing
3	10	2	5.18	81	2.347	2.33E-06	7.21E-04	Existing
4	10	2	5.16	81	2.337	2.32E-06	7.20E-04	Existing
5	10	2	6.08	81	2.754	2.02E-06	6.05E-04	Existing
6	10	2	6.07	81	2.750	2.04E-06	6.06E-04	Existing
7	10	3	24.39	81	11.049	1.93E-06	3.38E-03	Existing
8	10	3	24.39	81	11.049	1.93E-06	3.38E-03	Existing
9	11	3	22.57	81	10.224	2.96E-06	3.04E-03	Existing
10	11	3	22.57	81	10.224	2.96E-06	3.04E-03	Existing
11	11	8	4.53	81	2.052	4.04E-06	3.90E-04	Existing
12	11	8	4.53	81	2.052	3.91E-06	3.78E-04	Existing
13	11	8	4.52	81	2.048	3.90E-06	3.77E-04	Existing
14	11	8	4.68	81	2.120	3.90E-06	3.76E-04	Existing

Feeder	From Node	To Node	Length km	Capacity MVA	Cost M$	Loss coefficient M$/MVA²	G kV/MVA	Status
15	11	9	16.52	81	7.484	4.50E-06	2.06E-03	Existing
16	11	9	16.52	81	7.484	4.50E-06	2.06E-03	Existing
17	1	6	7.32	81	3.316	6.32E-06	6.10E-04	Existing
18	1	6	7.32	81	3.316	6.32E-06	6.10E-04	Existing
19	2	4	5.26	88	2.383	5.40E-06	5.14E-04	Existing
20	2	4	5.26	88	2.383	5.40E-06	5.14E-04	Existing
21	2	5	4.77	88	2.161	4.89E-06	4.66E-04	Existing
22	2	5	4.77	88	2.161	4.89E-06	4.66E-04	Existing
23	2	7	6.25	81	2.831	8.59E-06	8.18E-04	Existing
24	6	8	1.89	81	0.856	1.63E-06	1.58E-04	Existing
25	7	8	4.63	81	2.097	3.99E-06	3.86E-04	Existing
26	7	8	5.99	88	2.713	6.15E-06	5.85E-04	Existing
27	8	9	10.07	81	4.562	8.69E-06	8.40E-04	Existing
28	12	13	7.00	81	3.171	6.04E-06	5.84E-04	Potential
29	12	3	7.25	81	3.284	6.26E-06	6.04E-04	Potential
30	12	14	12.10	81	5.481	1.04E-05	1.01E-03	Potential
31	5	13	3.85	81	1.744	3.32E-06	3.21E-04	Potential
32	7	14	5.10	81	2.310	4.40E-06	4.25E-04	Potential
28	12	13	7.00	88	3.171	7.19E-06	6.84E-04	Potential
29	12	3	7.25	88	3.284	7.44E-06	7.09E-04	Potential
30	12	14	12.10	88	5.481	1.24E-05	1.18E-03	Potential
31	5	13	3.85	88	1.744	3.95E-06	3.76E-04	Potential
32	7	14	5.10	88	2.310	5.24E-06	4.99E-04	Potential

Case 1

It is the application of the basic model to a distribution system having a planning period of two years. Figure 1 shows the initial system, the routes proposed to build future feeders and the proposed location to build substation C. The proposed feeders are 630 mm^2 cable, 630 mm^2 cable XLPE and 2x630 mm^2 cable. The capacity of substation C is proposed to be 180 MVA. The costs of investment and energy losses are included in the objective function. Figures 2(a & b) show the optimal solution where the optimal feeders size selected is 630 mm^2 except that the feeder over route (7, 14) is 2x630 mm^2. The substation C and the feeder over route (12, 13) are required to be built in the second year. The optimal value of the objective function is 15.914435 million dollars.

Case 2

It is the application of the basic model to the system having a planning period of four years. There are only two possible feeder sizes (*i.e.*, 630 mm^2 and 630 mm^2 XLPE) in the proposed routes (shown in Fig. 1) and 180 MVA is the size of substation C. The cost of energy losses is not included in the objective function. Figures 3(a - d) show the optimal solution where the optimal feeder size is 630 mm^2 XLPE. The substation C and the feeder over route (12, 13) are required to be built in the third year. The feeder over route (12, 14) is required to be built in the fourth year. The optimal value of the objective function is 18.706650 million dollars.

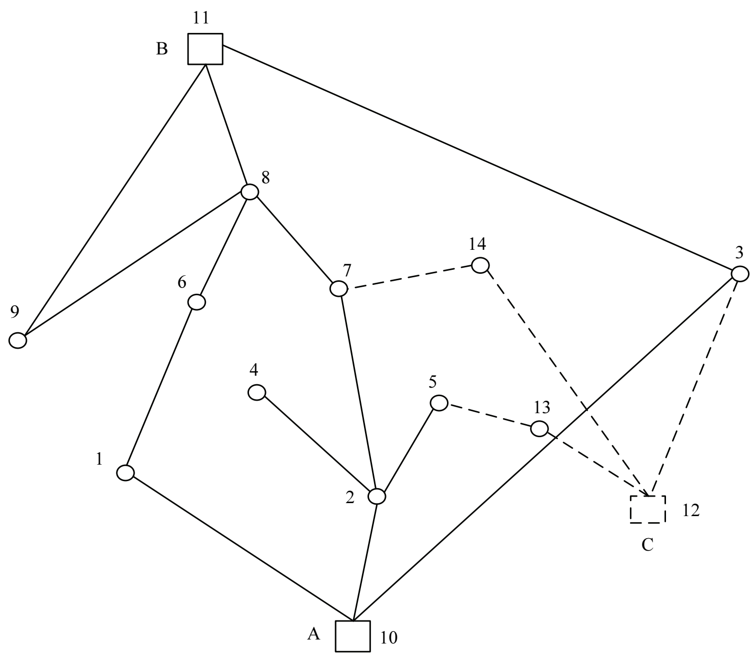


Fig. 1. Initial network and proposed routes.

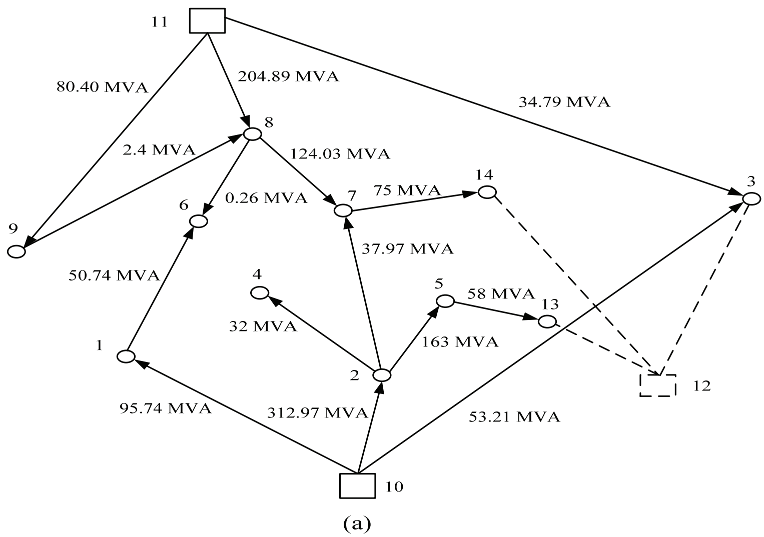


Fig. 2 (a). Optimal solution of case 1 for first year.

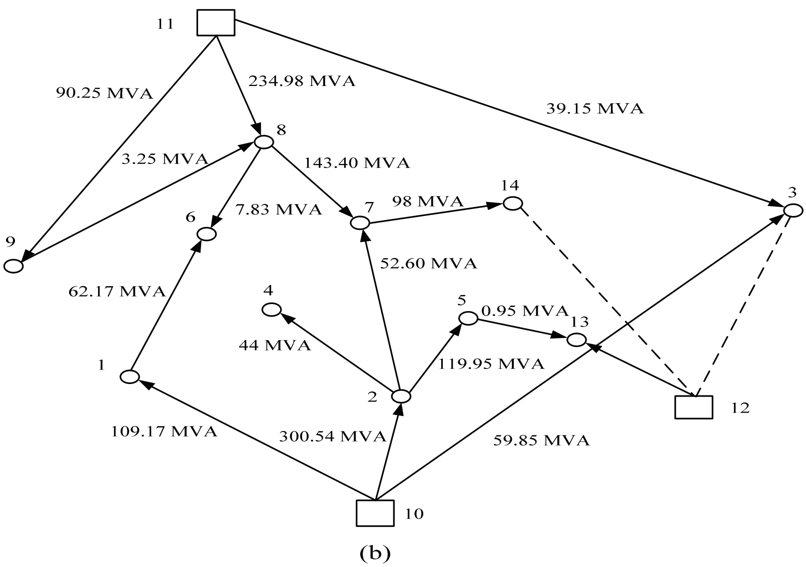
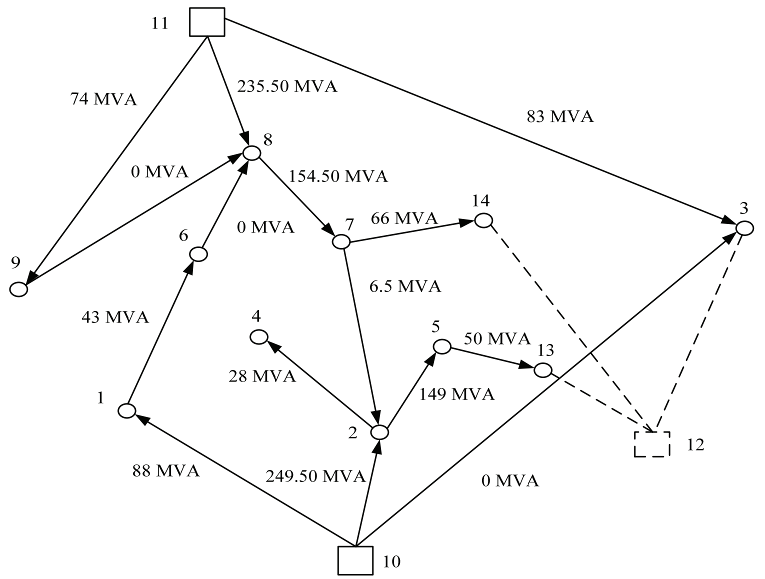
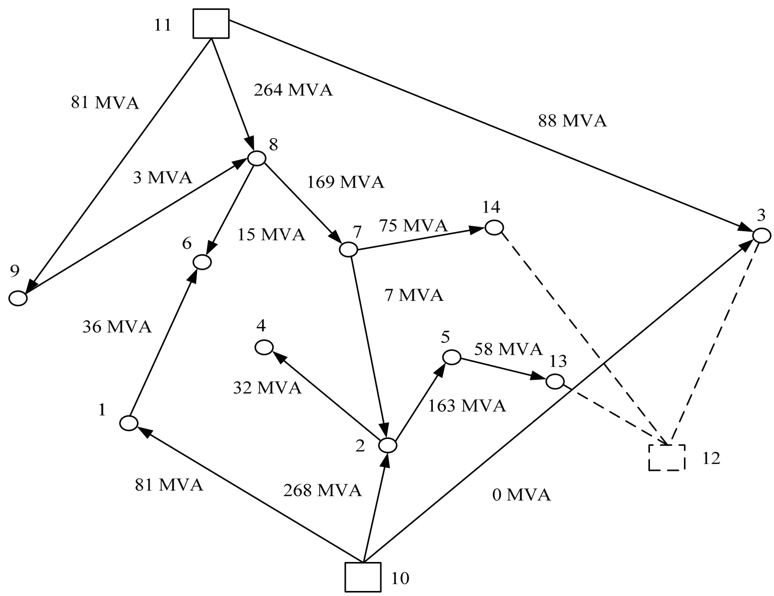


Fig. 2(b). Optimal solution of case 1 for second year.



(a)

Fig. 3(a). Optimal solution of case 2 for first year.



(b)

Fig. 3(b). Optimal solution of case 2 for second year.

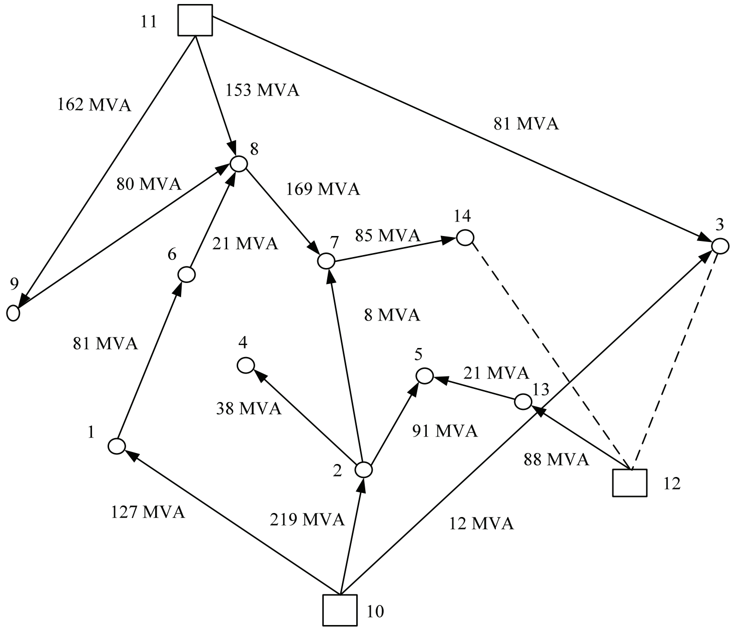


Fig. 3(c). Optimal solution of case 2 for third year.

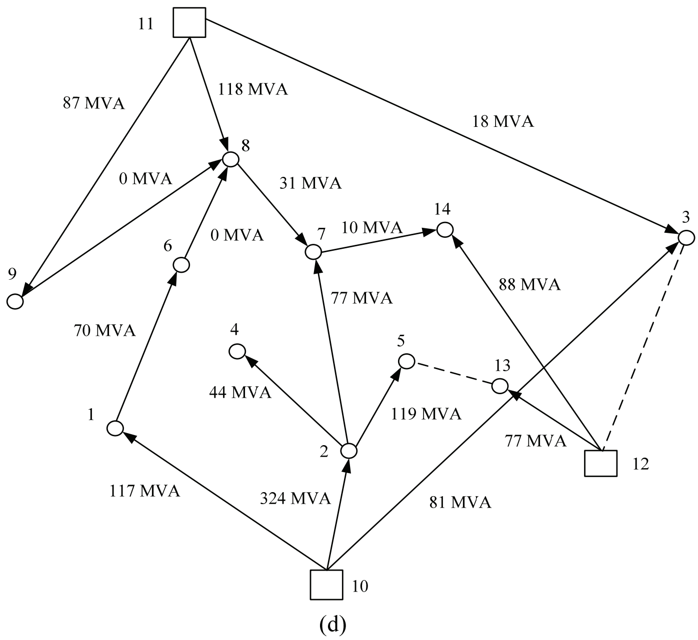


Fig. 3(d). Optimal solution of case 2 for fourth year.

Case 3

The system of two planning years is reconsidered including the voltage drop constraints. Only 630 mm^2 and 630 mm^2 XLPE for feeder sizes and 180 and 270 MVA for substation C are used as the possible sizes. Figures 4(a & b) show the optimal solution where the optimal feeder size is 630 mm^2. The optimal substation size is 180 MVA. The feeder over route (7, 14) is required to be built in the second year. The optimal value of the objective function is 17.502189 million dollars.

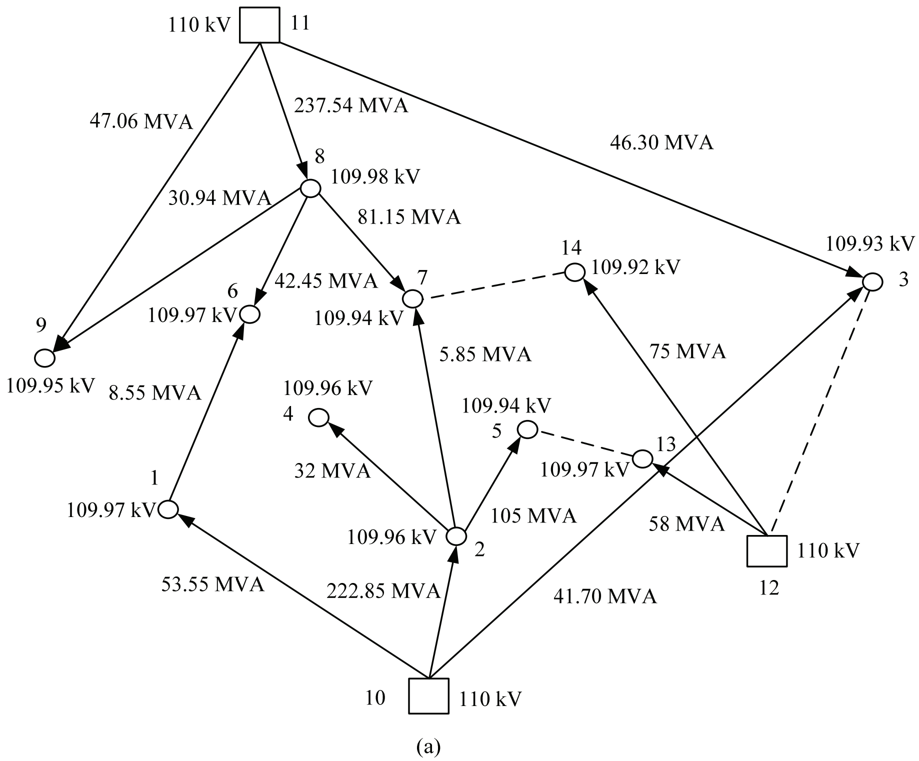
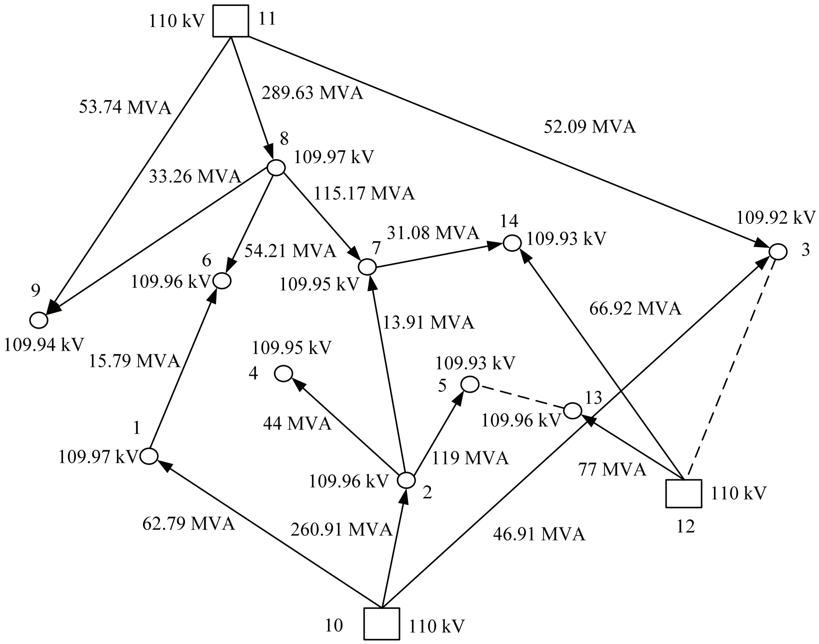


Fig. 4(a). Optimal solution of case 3 for first year.



(b)

Fig. 4(b). Optimal solution of case 3 for Second year.

Conclusions

The planning model takes into account the decision variables for substation and feeder building throughout the planning period simultaneously and provides the optimal solution for the sizing, location, and timing decisions.

The model has been applied to practical cases. The obtained solutions in this paper seem to indicate that the energy losses mainly determine the optimal feeder size, and the investment cost always dictates the optimal routing in the problems studied. Therefore, at least, the energy losses cost are recommended to be included together with the investment cost in the planning models. The results indicate that the more complex alternatives (e.g., more possible sizes for substations and feeders) can lead to less expensive distribution expansion in the optimal solution.

The results show that the voltage drop constraints increase the optimal expansion cost to more realistic amounts and affect the structural evolution of the power networks expansion throughout the time.

The planning model overcomes the limitations existing in previously reported power distribution planning models.

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تخطيط شبكات القدرة الكهربائية باستخدام برمجة الأعداد الصحيحة والمختلطة

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المستخلص. الهدف الرئيس من هذه الورقة هو بناء معادلات أنموذج تخطيط توزيع القدرة الكهربائية. وذلك لإيجاد القيم الأفضل: لأحجام محطات توزيع القدرة الكهربائية وزمن تنفيذ بنائها وموقعها بالنسبة للأحمال وكذلك اعتبار التوسع المستقبلي في أعداد كوابل التغذية. على أن يتم التعامل مع كل تلك المتغيرات مجتمعة. كما أن دالة التعريف للأنموذج تأخذ في الحساب الحصول على أقل تكلفة إستثمارية ممكنة وأقل مقادير للفقد في الطاقة خلال زمن التخطيط. وتتم عملية البحث عن القيم الأفضل تحت شروط محددة لايمكن تجاوزها وهي: قانون خرتشوف للتيار الكهربائي وسعة الحمل الكهربائي لمعدات التوزيع وكذلك شرط المحافظة على مستوى إنخفاض الجهد في الحدود المسموح بها، إضافة لبعض الشروط المنطقية الأخرى اللازمة لبرنامج المفاضلة ذي الأعداد الصحيحة والمختلطة.